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APPLICATIONS OF WASPAS METHOD AS A MULTI-CRITERIA DECISION-MAKING TOOL

Abstract. *The weighted aggregated sum product assessment (WASPAS) method is a unique combination of weighted sum model (WSM) and weighted product model (WPM). Because of its mathematical simplicity and capability to provide more accurate results as compared to WSM and WPM methods, it is now being widely accepted as an efficient decision-making tool. In this paper, its applicability is validated using five real time manufacturing related problems while selecting (a) a flexible manufacturing system, (b) a machine in a flexible manufacturing cell, (c) an automated guided vehicle, (d) an automated inspection system, and (e) an industrial robot. It is observed that for all these five problems, WASPAS method provides quite acceptable results. The optimal λ values for each of the considered problem are determined and the effects of varying λ values on ranking of the candidate alternatives in WASPAS method are also analyzed.*

Key words: *MCDM (Multiple Criteria Decision Making), WASPAS (Weighted Aggregated Sum Product Assessment), WSM (Weighted Sum Model); WPM (Weighted Product Model); manufacturing, ranking.*

JEL Classification: C02, C44, C61, C63, L6

1. Introduction

Multi-criteria decision-making (MCDM) methods are gaining importance as potential tools for analyzing and solving complex real time problems due to their inherent ability to evaluate different alternatives with respect to various criteria for possible selection of the best alternative. MCDM problems have several uniqueness such as presence of multiple non-commensurable and conflicting criteria, different units of measurement among the criteria, also presence of quite different alternatives. These decision-making problems describing multidimensional situations are being solved by various MCDM methods. The MCDM methods are primarily aimed at evaluating and ranking the available alternatives. There are several cases when different MCDM methods give different results (i.e. ranks of the same alternatives differ depending on the methods adopted). This can be attributed due to different mathematical artefacts employed by the considered methods. However, the problem of choosing an appropriate MCDM method in a particular situation still exists. Selection of MCDM methods based on various parameters has already been studied by the earlier researchers: Zavadskas *et al.* (2006), Antucheviciene *et al.* (2011), Simanaviciene and Ustinovicius (2012).

When a particular MCDM method is finally recommended for a specific application, it is observed that its solution accuracy and ranking performance are seriously influenced by the value of its control parameter. In this paper, the applicability and usefulness of weighted aggregated sum product assessment (WASPAS) method is explored while solving five problems from real time manufacturing environment. The considered problems are the selection of (a) a flexible manufacturing system, (b) a machine in a flexible manufacturing cell, (c) an automated guided vehicle, (d) an automated inspection system, and (e) an industrial robot. For each problem, the optimal λ values (control parameter for WASPAS method) are determined and their effects on the ranking of candidate alternatives are studied.

2. WASPAS method

The WASPAS method is a unique combination of two well-known MCDM approaches, i.e. weighted sum model (WSM) and weighted product model (WPM). Its application first requires development of a decision/evaluation matrix, $X = [x_{ij}]_{m\times n}$ where x_{ij} is the performance of i^{th} alternative with respect to j^{th} criterion, m is the number of alternatives and n is the number of criteria. To make the performance measures comparable and dimensionless, all the elements in the decision matrix are normalized using the following two equations:

$$\bar{x}_{ij} = \frac{x_{ij}}{\max_i x_{ij}} \text{ for beneficial criteria,} \qquad (1)$$

$$\bar{x}_{ij} = \frac{\min_i x_{ij}}{x_{ij}} \text{ for non-beneficial criteria,} \qquad (2)$$

where $\bar{x}_{ij}$ is the normalized value of x_{ij}.

In WASPAS method, a joint criterion of optimality is sought based on two criteria of optimality. The first criterion of optimality, i.e. criterion of a mean weighted success is similar to WSM method. It is a popular and well accepted MCDM approach applied for evaluating a number of alternatives with respect to a set of decision criteria. Based on WSM method (MacCrimon, 1968; Triantaphyllou and Mann, 1989), the total relative importance of i^{th} alternative is calculated as follows:

$$Q_i^{(1)} = \sum_{j=1}^{n} \bar{x}_{ij} w_j \ , \tag{3}$$

where w_j is weight (relative importance) of j^{th} criterion.

On the other hand, according to WPM method (Miller and Starr, 1969; Triantaphyllou and Mann, 1989), the total relative importance of i^{th} alternative is evaluated using the following equation:

$$Q_i^{(2)} = \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} \ . \tag{4}$$

A joint generalized criterion of weighted aggregation of additive and multiplicative methods is then proposed as follows (Saparauskas *et al.*, 2011;):

$$Q_i = 0.5 Q_i^{(1)} + 0.5 Q_i^{(2)} = 0.5 \sum_{j=1}^{n} \bar{x}_{ij} w_j + 0.5 \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} \ . \tag{5}$$

In order to have increased ranking accuracy and effectiveness of the decision-making process, in WASPAS method, a more generalized equation for determining the total relative importance of i^{th} alternative is developed (Zavadskas *et al.*, 2012; Zavadskas *et al.*, 2013a, b) as below:

$$Q_i = \lambda Q_i^{(1)} + (1-\lambda) Q_i^{(2)} = \lambda \sum_{j=1}^{n} \bar{x}_{ij} w_j + (1-\lambda) \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} , \ \lambda = 0, 0.1, \ldots, 1 \ . \tag{6}$$

The feasible alternatives are now ranked based on the Q values and the best alternative has the highest Q value. In Eq. (6), when the value of λ is 0, WASPAS method is transformed to WPM, and when λ is 1, it becomes WSM method. It has been applied for solving MCDM problems for increasing ranking accuracy and it has the capability to reach the highest accuracy of estimation (Bagocius *et al.*, 2013, 2014; Dejus and Antucheviciene, 2013; Hashemkhani Zolfani *et al*., 2013; Siozinyte and Antucheviciene, 2013; Staniunas *et al*., 2013; Chakraborty and Zavadskas, 2014).

For a given decision-making problem, the optimal values of λ can be determined while searching the following extreme function (Zavadskas *et al*., 2012):

$$\lambda = \frac{\sigma^2(Q_i^{(2)})}{\sigma^2(Q_i^{(1)}) + \sigma^2(Q_i^{(2)})} . \tag{7}$$

The variances $\sigma^2(Q_i^{(1)})$ and $\sigma^2(Q_i^{(2)})$ can be computed applying the equations as given below:

$$\sigma^2(Q_i^{(1)}) = \sum_{j=1}^{n} w_j^2 \sigma^2(\bar{x}_{ij}) , \tag{8}$$

$$\sigma^2(Q_i^{(2)}) = \sum_{j=1}^{n} \left(\frac{\prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} w_j}{\left(\bar{x}_{ij}\right)^{w_j} \left(\bar{x}_{ij}\right)^{(1-w_j)}} \right)^2 \sigma^2(\bar{x}_{ij}) . \tag{9}$$

The estimates of variances of the normalized initial criteria values are calculated as follows:

$$\sigma^2(\bar{x}_{ij}) = (0.05\bar{x}_{ij})^2 . \tag{10}$$

Variances of estimates of alternatives in WASPAS method depend of the variances of WSM and WPM approaches as well as on the value of λ. It may be worthwhile to compute the optimal values of λ and assure the maximum accuracy of estimation. It may also be important to study the effects of optimal λ values on the final ranking of the alternatives.

3. Illustrative examples

In order to justify the applicability and usefulness of WASPAS method as an effective decision-making tool, the following five illustrative examples are cited.

3.1. Example 1: FMS selection

A flexible manufacturing system (FMS) consists of computerized numerical control machines and/or robots, physically linked by a conveyance network to move parts and/or tools, and an overall effective computer control to create an integrated system. The reason the FMS is called ‘flexible’ is that it is capable of processing a variety of different part styles simultaneously at various workstations, and the mix of part styles and production quantities can be easily adjusted in response to changing demand patterns. Potential benefits of an FMS implementation include reduced inventory levels, manufacturing lead times, floor space, and setup and labor costs, in addition to higher flexibility, quality, speed of response and a longer useful life of the equipment over successive generations of products (Mondal and Chakraborty, 2011). Rao and Parnichkun (2009) applied a combinatorial mathematics-based approach to solve an FMS selection problem, consisting of seven criteria, i.e. percentage of reduction in labor cost (RLC), percentage of reduction in WIP (RWP), percentage of reduction in setup cost (RSC), increase in market response (IMR), increase in quality (IQ), capital and maintenance cost (CMC) (in thousand dollar), and floor space used (FSU) (in sq. ft.) and eight FMS alternatives. The original decision matrix for this FMS selection problem is shown in Table 1. Applying analytic hierarchy process (AHP), Rao and Parnichkun (2009) also determined the weights of the considered seven criteria as w_{RLC} = 0.1181, w_{RWP} = 0.1181, w_{RSC} = 0.0445, w_{IMR} = 0.1181, w_{IQ} = 0.2861, w_{CMC} = 0.2861 and w_{FSU} = 0.0445. Among these seven criteria, RLC, RWP, RSC, IMR and IQ are beneficial in nature and their higher values are desirable; on the other hand, CMC and FSU are non-beneficial attributes for which lower values are always preferable. Rao and Parnichkun (2009) observed the ranking of FMS alternatives as 3-4-7-2-5-6-1-8. While solving the same problem using WASPAS method, the decision matrix first needs to be linear normalized, as given in Table 2. Based on the results attained through WASPAS method-based analysis, the rank ordering of FMS alternatives is derived as 3-5-7-1-4-6-2-8 for a λ value of 0.5. When these two rank orderings are compared, an excellent Spearman’s rank correlation coefficient (r_s) of 0.9524 proves the suitability WASPAS method in solving complex decision-making problems. Table 3 exhibits the effects of changing values of λ on the ranking performance of WASPAS method. It is interesting to observe that for varying λ values, the positions of the top two and the worst FMS alternatives remain entirely unaltered, although the rankings of the intermediate alternatives slightly change. For a λ value of 0, the rank order of FMS alternatives is achieved as 3-4-7-1-5-6-2-8 (r_s = 0.9762), and for λ = 1, it is attained as 3-5-6-1-4-7-2-8 (r_s = 0.9405). Thus, it is observed that the overall ranking of eight FMS alternatives depends on λ value for which the optimal solution needs to be explored. In Table 4, the optimal values of λ for the considered alternatives are provided.

Table 1. Decision matrix for FMS selection problem (Rao and Parnichkun, 2009)

FMS	RLC	RWP	RSC	IMR	IQ	CMC	FSU
1	30	23	5	0.745	0.745	1500	5000
2	18	13	15	0.745	0.745	1300	6000
3	15	12	10	0.500	0.500	950	7000
4	25	20	13	0.745	0.745	1200	4000
5	14	18	14	0.255	0.745	950	3500
6	17	15	9	0.745	0.500	1250	5250
7	23	18	20	0.500	0.745	1100	3000
8	16	8	14	0.255	0.500	1500	3000

Table 2. Normalized data for FMS selection problem

FMS	RLC	RWP	RSC	IMR	IQ	CMC	FSU	$Q^{(1)}$	$Q^{(2)}$	Q
1	1	1	0.2500	1	1	0.6333	0.6000	0.8834	0.7991	0.8412
2	0.6000	0.5652	0.7500	1	1	0.7308	0.5000	0.8324	0.7657	0.7991
3	0.5000	0.5217	0.5000	0.6711	0.6711	1	0.4286	0.7440	0.6728	0.7084
4	0.8333	0.8696	0.6500	1	1	0.7917	0.7500	0.9211	0.8683	0.8947
5	0.4667	0.7826	0.7000	0.3423	1	1	0.8571	0.8596	0.7641	0.8118
6	0.5667	0.6522	0.4500	1	0.6711	0.7600	0.5714	0.7384	0.6825	0.7104
7	0.7667	0.7826	1	0.6711	1	0.8636	1	0.9133	0.8595	0.8864
8	0.5333	0.3478	0.7000	0.3423	0.6711	0.6333	1	0.6140	0.5495	0.5818

Table 3. Effect of λ on ranking performance of WASPAS method for Example 1

$\lambda = 0$	$\lambda =0.1$	$\lambda =0.2$	$\lambda =0.3$	$\lambda =0.4$	$\lambda =0.5$	$\lambda =0.6$	$\lambda =0.7$	$\lambda =0.8$	$\lambda =0.9$	$\lambda =1.0$
0.7991 (3)	0.8075 (3)	0.8159 (3)	0.8244 (3)	0.8328 (3)	0.8412 (3)	0.8497 (3)	0.8581 (3)	0.8665 (3)	0.8750 (3)	0.8834 (3)
0.7657 (4)	0.7724 (5)	0.7791 (5)	0.7857 (5)	0.7924 (5)	0.7991 (5)	0.8057 (5)	0.8124 (5)	0.8191 (5)	0.8258 (5)	0.8324 (5)
0.6728 (7)	0.6799 (7)	0.6870 (7)	0.6942 (7)	0.7013 (7)	0.7084 (7)	0.7155 (7)	0.7226 (6)	0.7297 (6)	0.7369 (6)	0.7440 (6)
0.8683 (1)	0.8736 (1)	0.8789 (1)	0.8842 (1)	0.8894 (1)	0.8947 (1)	0.9000 (1)	0.9053 (1)	0.9105 (1)	0.9158 (1)	0.9211 (1)
0.7641 (5)	0.7736 (4)	0.7832 (4)	0.7927 (4)	0.8023 (4)	0.8118 (4)	0.8214 (4)	0.8309 (4)	0.8405 (4)	0.8500 (4)	0.8596 (4)
0.6825 (6)	0.6881 (6)	0.6937 (6)	0.6993 (6)	0.7048 (6)	0.7104 (6)	0.7160 (6)	0.7216 (7)	0.7272 (7)	0.7328 (7)	0.7384 (7)
0.8595 (2)	0.8649 (2)	0.8703 (2)	0.8756 (2)	0.8810 (2)	0.8864 (2)	0.8918 (2)	0.8972 (2)	0.9025 (2)	0.9079 (2)	0.9133 (2)
0.5495 (8)	0.5560 (8)	0.5624 (8)	0.5689 (8)	0.5753 (8)	0.5818 (8)	0.5882 (8)	0.5947 (8)	0.6011 (8)	0.6076 (8)	0.6140 (8)

Table 4. Optimal λ values for Example 1

FMS	$\sigma^2(Q_i^{(1)})$	$\sigma^2(Q_i^{(2)})$	λ	Score
1	0.000422	0.000361	0.4609	0.8380
2	0.000408	0.000331	0.4482	0.7956
3	0.000363	0.000256	0.4137	0.7023
4	0.000457	0.000426	0.4827	0.8938
5	0.000490	0.000330	0.4027	0.8026
6	0.000295	0.000263	0.4716	0.7089
7	0.000461	0.000418	0.4753	0.8851
8	0.000218	0.000171	0.4395	0.5778

3.2. Example 2: Machine selection in FMC

Flexible manufacturing cells (FMC) are being used as a tool to implement flexible manufacturing processes to increase their competitiveness. While implementing an FMC, the decision makers often encounter the machine selection problem involving various attributes, like machine type, cost, number of machines, floor space and planned expenditures (Wang *et al*., 2000). Wang *et al*. (2000) considered some important constraints into the total purchasing cost, and also into the specifications of milling machine, lathe machine and robot, and developed ten possible alternative machine groups of FMC. The original decision matrix consisting of ten alternatives and four criteria, i.e. total purchasing cost (PC) (in \$), total floor space (FS) (in m^2), total number of machines in a machine group of the FMC (MN) and productivity (P) (in mm/min) is shown in Table 5. Among these four criteria, PC, FS and MN are non-beneficial attributes requiring lower values. Using a fuzzy multi-attribute decision-making approach, Wang *et al*. (2000) identified the dominant alternatives as 4, 5 and 3 for final selection. On the other hand, Rao (2007) determined the priority weights of the four criteria as w_{PC} = 0.467, w_{FS} = 0.160, w_{MN} = 0.095 and w_P = 0.278, and applied AHP method to evaluate the ranking of the alternatives as 6-10-8-1-5-9-7-3-2-4. While solving the same machine selection problem in FMC using WASPAS method, the decision matrix of Table 5 is first normalized as given in Table 6. It is observed that for a λ value of 0.5, alternative 4 tops the ranking list followed by alternative 9, whereas alternative 2 is the worst preferred choice. In this case, a total ranking preorder of the considered alternatives as 6-10-8-1-5-9-7-3-2-4 is achieved. It is quite interesting to observe that in WASPAS method, an exact rank agreement occurs with the observation of Wang *et al*. (2000). Table 7 exhibits the effects of varying λ values on the ranking performance of WASPAS method. It is noted that for this machine selection problem in FMC, WASPAS method is extremely robust being totally unaffected by the changing λ values. Table 8 shows the optimal λ values for this problem.

Table 5. Decision matrix for machine selection problem in FMC (Wang *et al*., 2000)

Alternative	PC	FS	MN	P
1	581818	54.49	3	5500
2	595454	49.73	3	4500
3	586060	51.24	3	5000
4	522727	45.71	3	5800
5	561818	52.66	3	5200
6	543030	74.46	4	5600
7	522727	75.42	4	5800
8	486970	62.62	4	5600
9	509394	65.87	4	6400
10	513333	70.67	4	6000

Table 6. Normalized decision matrix for Example 2

Alternative	PC	FS	MN	P	$Q^{(1)}$	$Q^{(2)}$	Q
1	0.8370	0.8389	1	0.8594	0.8590	0.8578	0.8584
2	0.8178	0.9192	1	0.7031	0.8194	0.8144	0.8169
3	0.8309	0.8921	1	0.7812	0.8430	0.8408	0.8419
4	0.9316	1	1	0.9062	0.9420	0.9413	0.9417
5	0.8668	0.8680	1	0.8125	0.8645	0.8632	0.8638
6	0.8968	0.6139	0.75	0.8750	0.8315	0.8241	0.8278
7	0.9316	0.6061	0.75	0.9062	0.8552	0.8454	0.8503
8	1	0.7300	0.75	0.8750	0.8983	0.8915	0.8949
9	0.9560	0.6940	0.75	1	0.9067	0.8987	0.9027
10	0.9486	0.6468	0.75	0.9375	0.8784	0.8697	0.8740

Table 7. Ranking performance of WASPAS method for Example 2

λ = 0	λ =0.1	λ =0.2	λ =0.3	λ =0.4	λ =0.5	λ =0.6	λ =0.7	λ =0.8	λ =0.9	λ =1.0
0.8578 (6)	0.8580 (6)	0.8581 (6)	0.8582 (6)	0.8583 (6)	0.8584 (6)	0.8585 (6)	0.8586 (6)	0.8588 (6)	0.8589 (6)	0.8590 (6)
0.8144 (10)	0.8149 (10)	0.8154 (10)	0.8159 (10)	0.8164 (10)	0.8169 (10)	0.8174 (10)	0.8179 (10)	0.8184 (10)	0.8189 (10)	0.8194 (10)
0.8408 (8)	0.8410 (8)	0.8412 (8)	0.8414 (8)	0.8417 (8)	0.8419 (8)	0.8421 (8)	0.8423 (8)	0.8425 (8)	0.8427 (8)	0.8430 (8)
0.9413 (1)	0.9414 (1)	0.9415 (1)	0.9415 (1)	0.9416 (1)	0.9417 (1)	0.9417 (1)	0.9418 (1)	0.9419 (1)	0.9419 (1)	0.9420 (1)
0.8632 (5)	0.8633 (5)	0.8634 (5)	0.8636 (5)	0.8637 (5)	0.8638 (5)	0.8640 (5)	0.8641 (5)	0.8643 (5)	0.8644 (5)	0.8645 (5)
0.8241 (9)	0.8249 (9)	0.8256 (9)	0.8263 (9)	0.8271 (9)	0.8278 (9)	0.8286 (9)	0.8293 (9)	0.8300 (9)	0.8308 (9)	0.8315 (9)
0.8454 (7)	0.8464 (7)	0.8474 (7)	0.8484 (7)	0.8493 (7)	0.8503 (7)	0.8513 (7)	0.8523 (7)	0.8532 (7)	0.8542 (7)	0.8552 (7)
0.8915	0.8922	0.8929	0.8936	0.8942	0.8949	0.8956	0.8963	0.8969	0.8976	0.8983

(3)	(3)	(3)	(3)	(3)	(3)	(3)	(3)	(3)	(3)	(3)
0.8987 (2)	0.8995 (2)	0.9003 (2)	0.9011 (2)	0.9019 (2)	0.9027 (2)	0.9035 (2)	0.9043 (2)	0.9051 (2)	0.9059 (2)	0.9067 (2)
0.8697 (4)	0.8706 (4)	0.8714 (4)	0.8723 (4)	0.8732 (4)	0.8740 (4)	0.8749 (4)	0.8758 (4)	0.8766 (4)	0.8775 (4)	0.8784 (4)

Table 8. Optimal λ values for Example 2

Alternative	$\sigma^2(Q_i^{(1)})$	$\sigma^2(Q_i^{(2)})$	λ	Score
1.	0.000592	0.000607	0.50616	0.8584
2.	0.000537	0.000547	0.50479	0.8169
3.	0.000568	0.000583	0.50669	0.8419
4.	0.000718	0.000731	0.50434	0.9417
5.	0.000608	0.000615	0.50275	0.8639
6.	0.000623	0.000560	0.47341	0.8276
7.	0.000668	0.000590	0.46882	0.8500
8.	0.000740	0.000656	0.46981	0.8947
9.	0.000735	0.000666	0.47549	0.9025
10.	0.000700	0.000624	0.47134	0.8738

3.3. Example 3: AGV system selection

Nowadays, automated guided vehicles (AGVs) play an important role in material handling in manufacturing organizations where FMS and computer integrated manufacturing are employed. The AGV system is a driverless and programmed vehicle, used to transfer load from one workstation to another. Selection of AGV systems not only increases flexibility of manufacturing systems but also enhances flow in materials handling. Other benefits of AGV systems include labor cost saving, flexible material handling, effective inventory control, quality assurance, utilization of space and flexible routing (Maniya and Bhatt, 2011). Hence, to do investments in a proper AGV system is an important task for a decision maker. Maniya and Bhatt (2011) solved an AGV system selection problem, consisting of eight alternatives and six criteria applying a modified grey relational analysis (M-GRA) and AHP method. The six criteria are controllability (C_1), accuracy (C_2), cost (C_3), range (C_4), reliability (C_5) and flexibility (C_6). The AHP method was used to determine the relative importance of AGV selection attributes as w_{C1} = 0.346, w_{C2} = 0.168, w_{C3} = 0.0584, w_{C4} = 0.073, w_{C5} = 0.063 and w_{C6} = 0.293, and M-GRA method was applied to compute the AGV selection utility index. Amongst the six evaluation criteria, cost is the sole non-beneficial attribute. The original qualitative decision matrix is given in Table 9, which is subsequently converted into a quantitative decision matrix of Table 10 while applying a suitable fuzzy conversion scale (Rao, 2007). Using the combined approach, Maniya and Bhatt (2011) ranked the AGV system alternatives as A_4-A_6-A_8-A_3-A_1-A_2-A_7-A_5. While solving this AGV system selection problem using WASPAS method for a λ value of 0.5, the ranking preorder of the considered eight alternatives is observed as A_3-A_5-A_8-A_2-A_1-A_4-A_7-A_6 from Table 11. A good Spearman's rank correlation of 0.9048 exists between the rankings of AGV system alternatives as obtained using WASPAS method and those by Maniya and Bhatt (2011). In Table 12, the ranking performance of WASPAS method for varying λ values is provided. It is observed from this table that for lower λ values, the ranking of the alternatives is A_3-A_5-A_8-A_2-A_1-A_4-A_7-A_6, and for higher λ values, it is A_3-A_5-A_8-A_2-A_1-A_4-A_6-A_7. The optimal λ values are shown in Table 13.

Table 9. Decision matrix for AGV system selection problem (Maniya and Bhatt, 2011)

Alternative	C_1	C_2	C_3	C_4	C_5	C_6
A_1	High	Average	Above average	Average	High	Below average
A_2	Low	High	High	High	Average	Average
A_3	Low	Low	High	Low		High
A_4	Below average	High	Low	Average	Average	High
A_5	High	Average	Low	Above average	Below average	Average
A_6	Average	Average	High	Low	Above average	Above average
A_7	Low	Below average	High	Low	High	High
A_8	Low	Average	Above average	Average	Average	Above average

Table 10. Quantitative decision matrix for AGV system selection problem

Alternative	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.895	0.495	0.695	0.495	0.895	0.295
A_2	0.115	0.895	0.895	0.895	0.495	0.495
A_3	0.115	0.115	0.895	0.115	0.695	0.895
A_4	0.295	0.895	0.115	0.495	0.495	0.895
A_5	0.895	0.495	0.115	0.695	0.295	0.495
A_6	0.495	0.495	0.895	0.115	0.695	0.695
A_7	0.115	0.295	0.895	0.115	0.895	0.895
A_8	0.115	0.495	0.695	0.495	0.495	0.695

Table 11. Normalized decision matrix for AGV system selection problem

Alternative	C_1	C_2	C_3	C_4	C_5	C_6	$Q^{(1)}$	$Q^{(2)}$	Q
A_1	1	0.5531	0.1655	0.5531	1	0.3296	0.6485	0.5638	0.6062
A_2	0.1285	1	0.1285	1	0.5531	0.5531	0.4898	0.3532	0.4215
A_3	0.1285	0.1285	0.1285	0.1285	0.7765	1	0.4248	0.2618	0.3433
A_4	0.3296	1	1	0.5531	0.5531	1	0.7087	0.6284	0.6685
A_5	1	0.5531	1	0.7765	0.3296	0.5531	0.7368	0.6967	0.7167
A_6	0.5531	0.5531	0.1285	0.1285	0.7765	0.7765	0.5776	0.5147	0.5462
A_7	0.1285	0.3296	0.1285	0.1285	1	1	0.4727	0.3116	0.3921
A_8	0.1285	0.5531	0.1655	0.5531	0.5531	0.7765	0.4498	0.3433	0.3965

Table 12. Effect of λ on ranking performance of WASPAS method for Example 3

$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$	$\lambda = 1.0$
0.5638 (3)	0.5723 (3)	0.5808 (3)	0.5892 (3)	0.5977 (3)	0.6062 (3)	0.6145 (3)	0.6231 (3)	0.6316 (3)	0.6401 (3)	0.6485 (3)
0.3532 (5)	0.3669 (5)	0.3805 (5)	0.3942 (5)	0.4079 (5)	0.4215 (5)	0.4352 (5)	0.4489 (5)	0.4625 (5)	0.4762 (5)	0.4898 (5)
0.2619 (8)	0.2781 (8)	0.2944 (8)	0.3107 (8)	0.3270 (8)	0.3433 (8)	0.3596 (8)	0.3759 (8)	0.3922 (8)	0.4085 (8)	0.4248 (8)
0.6284 (2)	0.6364 (2)	0.6445 (2)	0.6525 (2)	0.6605 (2)	0.6685 (2)	0.6766 (2)	0.6846 (2)	0.6926 (2)	0.7006 (2)	0.7087 (2)
0.6967 (1)	0.7007 (1)	0.7047 (1)	0.7087 (1)	0.7127 (1)	0.7167 (1)	0.7208 (1)	0.7248 (1)	0.7288 (1)	0.7328 (1)	0.7368 (1)
0.5147 (4)	0.5210 (4)	0.5273 (4)	0.5336 (4)	0.5399 (4)	0.5462 (4)	0.5525 (4)	0.5587 (4)	0.5650 (4)	0.5713 (4)	0.5776 (4)
0.3116 (7)	0.3277 (7)	0.3438 (7)	0.3599 (7)	0.3760 (7)	0.3921 (7)	0.4083 (6)	0.4244 (6)	0.4405 (6)	0.4566 (6)	0.4727 (6)
0.3433 (6)	0.3539 (6)	0.3646 (6)	0.3752 (6)	0.3859 (6)	0.3965 (6)	0.4072 (7)	0.4178 (7)	0.4285 (7)	0.4391 (7)	0.4498 (7)

Table 13. Optimal λ values for Example 3

Alternative	$\sigma^2(Q_i^{(1)})$	$\sigma^2(Q_i^{(2)})$	λ	Score
A_1	0.000358	0.000196	0.3534	0.5937
A_2	0.000158	0.000077	0.3278	0.3980
A_3	0.000227	0.000042	0.1568	0.2874
A_4	0.000333	0.000243	0.4220	0.6623
A_5	0.000404	0.000299	0.4253	0.7138
A_6	0.000249	0.000163	0.3961	0.5396
A_7	0.000238	0.000060	0.2012	0.3440
A_8	0.000163	0.000073	0.3079	0.3761

3.4. Example 4: Automated inspection system selection

Automated inspection systems, like coordinate measure machine (CMM), automated visual inspection (AVI) system, computer vision system etc. are now being extensively used in FMS to monitor the quality of manufacturing parts/components. Pandey and Kengpol (1995) considered an automated inspection system selection problem in which the performance of four such systems was evaluated based on 12 evaluation criteria, i.e. accuracy (A), volumetric performance (V), repeatability (R), resolution (S), maintainability (M), reliability (L), initial cost (I), operation cost (O), throughput rate (T), environmental factor requirement (E) and flexibility in software interface (F). The four alternatives were CMM1 (USA) (A), CMM2 (Japan) (B), AVI (USA) (C) and Laser scan (Japan) (D). Using PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) method, Pandey and Kengpol (1995) identified CMM1 (USA) as the best choice, whereas, AVI (USA) was the worst preferred automated inspection system. Rao (2007) also solved the same problem and determined the criteria weights as w_A = 0.2071, w_V = 0.0858, w_R = 0.2071, w_S = 0.0518, w_M = 0.0325, w_L = 0.0518, w_I = 0.0858, w_O = 0.0325, w_T = 0.1376, w_E = 0.0219 and w_F = 0.0858, which are also used here for the subsequent analyses. The decision matrix for this automated inspection system selection problem is given in Table 14, whereas, Table 15 shows the corresponding normalized decision matrix.

Table 14. Decision matrix for automated inspection system selection problem (Pandey and Kengpol, 1995)

Attribute	A	B	C	D
Accuracy (A)	90	80	60	75
Volumetric performance (V)	80	70	50	70
Repeatability I	80	80	50	70
Resolution (S)	70	70	80	60
Maintainability (M)	60	60	80	70
Reliability (L)	85	80	70	70
Initial cost (I)	40	30	20	25
Operation cost (O)	2	7	1	4
Throughput rate (T)	70	70	80	80
Environmental factor requirement I	80	80	60	70
Flexibility in software interface (F)	80	60	60	70

Table 15. Normalized decision matrix for automated inspection system selection problem

Attribute	A	B	C	D
A	1	0.8889	0.6667	0.8333
V	1	0.8750	0.6250	0.8750
R	1	1	0.6250	0.8750
S	0.8750	0.8750	1	0.7500
M	0.7500	0.7500	1	0.8750
L	1	0.9412	0.8235	0.8235
I	0.5000	0.6667	1	0.8000
O	0.5000	0.1428	1	0.2500
T	0.8750	0.8750	1	1
E	0.7500	0.7500	1	0.8571
F	1	0.7500	0.7500	0.8750
$Q^{(1)}$	0.9033	0.8477	0.7902	0.8470
$Q^{(2)}$	0.8843	0.8166	0.7738	0.8309
Q	0.8938	0.8321	0.7820	0.8390

From the results of WASPAS method-based analysis of Table 16, it is found that for a λ value of 0.5, CMM1 (USA) tops in the ranking list of the alternatives and AVI (USA) is the worst preferred automated inspection system. These findings exactly corroborate with those of Pandey and Kengpol (1995). The ranking performance of WASPAS method for changing values of λ is also provided in Table 16. Table 17 provides the optimal values of λ for this automated inspection system selection problem.

Table 16. Rankings of automated inspection systems for changing λ values

$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$	$\lambda = 1.0$
0.8843 (1)	0.8862 (1)	0.8881 (1)	0.8900 (1)	0.8919 (1)	0.8938 (1)	0.8957 (1)	0.8976 (1)	0.8995 (1)	0.9014 (1)	0.9033 (1)
0.8166 (3)	0.8197 (3)	0.8228 (3)	0.8259 (3)	0.8290 (3)	0.8321 (3)	0.8353 (3)	0.8384 (3)	0.8415 (3)	0.8446 (3)	0.8477 (2)
0.7738 (4)	0.7755 (4)	0.7771 (4)	0.7788 (4)	0.7804 (4)	0.7820 (4)	0.7837 (4)	0.7853 (4)	0.7870 (4)	0.7886 (4)	0.7902 (4)
0.8309 (2)	0.8325 (2)	0.8341 (2)	0.8358 (2)	0.8374 (2)	0.8390 (2)	0.8406 (2)	0.8422 (2)	0.8438 (2)	0.8454 (2)	0.8470 (3)

Table 17. Optimal λ values for Example 4

Alternative	$\sigma^2(Q_i^{(1)})$	$\sigma^2(Q_i^{(2)})$	λ	Score
A	0.000307	0.000263	0.4620	0.8931
B	0.000274	0.000225	0.4504	0.8306
C	0.000191	0.000202	0.5142	0.7822
D	0.000255	0.000233	0.4768	0.8386

3.5. Example 5: Robot selection problem

Agrawal *et al.* (1991) considered an industrial robot selection problem consisting of five alternatives and four criteria, i.e. load capacity (LC) (in kg), repeatability error (R) (in mm), vertical reach (VR) (in cm) and degrees of freedom (DF), and solved it using TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method. The decision matrix for this industrial robot selection problem is provided in Table 18. Rao and Padmanabhan (2009) also solved the same robot selection problem applying digraph and matrix approaches, and determined the ranking order of the alternative robots as 3-2-1-4-5. Using AHP method, the weights of the four criteria were determined as w_{LC} = 0.096325, w_{R} = 0.557864, w_{VR} = 0.096325 and w_{DF} = 0.249486, which are used here for the subsequent analyses. While applying WASPAS method for this robot selection problem, the decision matrix is first normalized, as given in Table 19. From this table, the ranking list of the robot alternatives is achieved as 3-2-1-4-5 for a λ value of 0.5. It is interesting to note that the ranking of the alternative robots exactly matches with that as obtained by Rao and Padmanabhan (2009). The ranking performance of WASPAS for varying λ values is exhibited in Table 20. It is observed that the last two positions of robots in the ranking list are affected by the changing λ values. The optimal λ values are determined in Table 21.

Table 18. Decision matrix for robot selection problem (Agrawal *et al.*, 1991)

Alternative	LC	R	VR	DF
Robot 1	60	0.4	125	5
Robot 2	60	0.4	125	6
Robot 3	68	0.13	75	6
Robot 4	50	1	100	6
Robot 5	30	0.6	55	5

Table 19. Normalized decision matrix for robot selection problem

Alternative	LC	R	VR	DF	$Q^{(1)}$	$Q^{(2)}$	Q
Robot 1	0.8823	0.3250	1	0.8333	0.5705	0.5043	0.5374
Robot 2	0.8823	0.3250	1	1	0.6121	0.5278	0.5700
Robot 3	1	1	0.6	1	0.9615	0.9520	0.9567
Robot 4	0.7353	0.1300	0.8	1	0.4699	0.3044	0.3872
Robot 5	0.4412	0.2167	0.44	0.8333	0.4136	0.3476	0.3806

Table 20. Rankings of robots for varying λ values

$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$	$\lambda = 1.0$
0.5043 (3)	0.5109 (3)	0.5176 (3)	0.5242 (3)	0.5308 (3)	0.5374 (3)	0.5440 (3)	0.5507 (3)	0.5573 (3)	0.5639 (3)	0.5705 (3)
0.5278 (2)	0.5362 (2)	0.5446 (2)	0.5531 (2)	0.5615 (2)	0.5700 (2)	0.5784 (2)	0.5868 (2)	0.5952 (2)	0.6037 (2)	0.6121 (2)
0.9520 (1)	0.9529 (1)	0.9534 (1)	0.9548 (1)	0.9558 (1)	0.9567 (1)	0.9577 (1)	0.9586 (1)	0.9596 (1)	0.9605 (1)	0.9615 (1)
0.3044 (5)	0.3210 (5)	0.3375 (5)	0.3541 (5)	0.3706 (5)	0.3872 (4)	0.4037 (4)	0.4202 (4)	0.4368 (4)	0.4533 (4)	0.4699 (4)
0.3476 (4)	0.3542 (4)	0.3608 (4)	0.3674 (4)	0.3740 (4)	0.3806 (5)	0.3872 (5)	0.3938 (5)	0.4004 (5)	0.4070 (5)	0.4136 (5)

Table 21. Optimal λ values for Example 5

Alternative	$\sigma^2(Q_i^{(1)})$	$\sigma^2(Q_i^{(2)})$	λ	Score
Robot 1	0.000231	0.000249	0.5185	0.5386
Robot 2	0.000279	0.000273	0.4945	0.5695
Robot 3	0.000965	0.000888	0.4792	0.9565
Robot 4	0.000196	0.000091	0.3165	0.3568
Robot 5	0.000154	0.000118	0.4354	0.3764

4. Conclusions

In this paper, the applicability and usefulness of WASPAS method as a decision-making tool is validated using five demonstrative examples. It is observed that for all the problems, the rankings as attained by WASPAS method closely match with those derived by the past researchers. For each considered problem, the optimal λ values for the alternatives are computed. The ranking performance of WASPAS method with respect to changing λ values is also studied. The robustness of WASPAS method is proved which will help in its widespread application as an efficient MCDM tool. As it is based on simple and sound mathematics, being quite comprehensive in nature, it can be successfully applied to any decision- making problem.

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